AN EMPIRICAL MODEL FOR PHASE DIFFERENCE STATISTICS OF ROUGH SURFACES

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ABSTRACT

Polarimetric radar measurements were conducted for bare soil surfaces under a variety of roughness and moisture conditions at 1.5, 4.75, and 9.5 GHz at incidence angles ranging from 20° to 70°. Using a laser profile meter and dielectric probes, a complete and accurate set of ground truth data were collected for each surface conditions, from which accurate measurement were made of the rms height, correlation length, and dielectric constant. Based on the experimental observations, an empirical model is developed for the degree of correlation α, which is a measure of the width of the probability density function of the co-polarized phase angle $\phi_c(=\phi_{hh}-\phi_{vv})$ for a distributed target. The validity of the proposed model is checked by comparing the results based on the empirical model with an independent data set measured by a different radar system.

1. INTRODUCTION

In the past two decades extensive research has been conducted towards relating the physical parameters of rough surfaces (rms height, correlation length, and moisture content) to their co- and cross-polarized backscattering coefficients. Polarimetric radars, however, provide two additional independent pieces of information, namely, the co- and cross-polarized phase differences.

So far the utilization of polarimetric data has been limited to the study of polarization signature of distributed targets. Although the polarization signature can be used to infer some qualitative characteristics of a target, they cannot provide quantitative information about the biophysical parameters of the target. Recently a technique was developed that extracts the phase difference statistics from the Mueller matrix of distributed targets [1]. It is shown that the probability density function of the co-polarized phase difference is completely specified by only two factors (α, ζ). In order to utilize all information available from polarimetric data, relationships between these parameters and the physical parameters of the target must be established.

Due to inadequacy of existing theoretical models, such relationships are not available. Using experimental data, an attempt has been made to establish relations empirically between the co-polarized phase parameters and the roughness and the dielectric constant of rough surfaces.

2. THEORY

The reciprocity theorem mandates symmetry of the scattering matrix of passive targets in backscattering direction, *i.e.*,

$$\mathbf{S} = e^{i\phi_{vv}} \begin{bmatrix} |S_{vv}| & |S_{vh}|e^{i\phi_x} \\ |S_{vh}|e^{i\phi_x} & |S_{hh}|e^{i\phi_c} \end{bmatrix} \tag{1}$$

where

$$\phi_x = \phi_{vh} - \phi_{vv} \tag{2}$$

$$\phi_c = \phi_{hh} - \phi_{vv} \tag{3}$$

We shall henceforth refer to ϕ_x and ϕ_c as cross- and co-polarized phase angle, respectively.

Experimental data acquired by polarimetric radars have shown that the co-polarized phase angle ϕ_c, in addition to the magnitude of the scattering matrix, contains information about the distributed target; that is, the probability density function (PDF) of ϕ_c is strongly dependent upon the target parameters. Figures 1 (a), (b), and (c) show the PDF of ϕ_c for a bare soil surface (rms height $s = 0.4cm$, correlation length $l = 8.4cm$, moisture content $m_v = 0.29$) at incidence angle of 30° at 1.5, 4.75, and 9.5 GHz, respectively. The cross-polarized phase angle ϕ_x is uniformly distributed over $[0, 2\pi]$ for almost all distributed targets as shown in Fig. 1 (d). Hence, ϕ_x usually does not contain any information about the target parameters.

It has been shown that the PDF of ϕ_c, $f(\phi_c)$, can be characterized completely by two parameters; the degree of correlation α which is a measure of the width of the PDF and the polarized-phase difference ζ which is the value of ϕ_c at the maximum of the PDF [1]. The $f(\phi_c)$ is given by

$$f(\phi_c) = \frac{1-\alpha^2}{2\pi[1-\alpha^2\cos^2(\phi_c-\zeta)]}\left\{1 + \frac{\alpha\cos(\phi_c-\zeta)}{\sqrt{1-\alpha^2\cos^2(\phi_c-\zeta)}} \times \left[\frac{\pi}{2} + \tan^{-1}\frac{\alpha\cos(\phi_c-\zeta)}{\sqrt{1-\alpha^2\cos^2(\phi_c-\zeta)}}\right]\right\} \tag{4}$$

with

$$\alpha = \frac{1}{2}\sqrt{\frac{(M_{33}+M_{44})^2 + (M_{34}-M_{43})^2}{M_{11}\,M_{22}}}\,, \tag{5}$$

$$\zeta = \tan^{-1}\left[\frac{M_{34}-M_{43}}{M_{33}+M_{44}}\right]\,, \tag{6}$$

where M_{ij} are the elements of the Mueller matrix. Fig. 2 shows plots of $f(\phi_c)$ for various values of α, with ζ held constant.

3. EXPERIMENTS

Using a truck-mounted polarimetric scatterometer, backscatter measurements were conducted for four bare-soil fields with different surface roughnesses. Each field was measured under two different moisture conditions, relatively wet and relatively dry, at 1.5, 4.75, and 9.5 GHz at incidence angles extending from 20° to 70°, in 10° steps.

In addition to the radar backscatter data, an accurate ground truth data were obtained. The surface height profiles and the dielectric constants of the soil surfaces were measured by a laser profile meter and C-band field portable dielectric probes, respectively. The surface roughness parameters such as rms height s, autocorrelation function $\rho(x)$, and correlation length l are calculated from the measured surface height profiles. The volumetric moisture contents m_v of the soil were obtained from the measured dielectric constants at C-band [2]. The ground truth data were summarized in Table 2 and 3 in [2].

Accurate calibration of the radar data was achieved using the differential Mueller matrix calibration technique (DMCT) [3]. In this method the polarimetric response of a sphere over the entire main lobe of the antenna is measured to characterize the error parameters required for calibration. It is shown that the statistics of the polarization phase differences can be measured very accurately using this calibration technique [3].

The measurement shows that the degree of correlation α depends on surface roughness, moisture content, frequency, and incidence angle. Figure 3 (a) shows that the α for a smooth surface (rms height $s = 0.4cm$ and f=4.75 GHz) is approximately 1.0 (PDF of ϕ_c is a delta function) at incidence angles $\theta \leq 30°$ and decreases rapidly as θ increases. The degree of correlation for a rough surface ($s = 3.0cm$ and f=4.75 GHz) has a lower value than the degree of correlation for a smooth surface, and remains constant ($\alpha \approx 0.8$) for incidence angles $30° \leq \theta \leq 70°$. Figure 3 (b) shows that the α of a relatively dry surface ($m_v = 0.15$) is higher than that of a relatively wet surface ($m_v = 0.31$). The measurement shows that the polarized-phase difference ζ is about 0° at $\theta < 30°$ and increases as incidence angle θ increases. The rate of change of ζ with incidence angle depends on both the roughness and the moisture contents of the soil surface.

4. EMPIRICAL MODEL

An empirical model for the degree of correlation α is developed using the radar backscatter data measured from bare soil surfaces.

After analyzing the measured data for α , we found that α at low incidence angles ($\theta \leq 30°$) is between 1.0 and 0.8 depending on the roughness ($\alpha \approx 1.0$ at $ks < 0.4$, $\alpha \approx 0.8$ at $ks > 2.5$). The rate of change of α with incidence angle depends on both the roughness (Fig. 3 (a)) and the moisture contents of the soil surface. It was found that the angular rate of change of α is small for very smooth ($ks < 0.2$) and very rough ($ks > 2.5$) surfaces. For intermediately rough surfaces, however, the rate of change of α with incidence angle is large. It was also found that the degree of correlation is inversely proportional to moisture content as shown in Fig. 3 (b) and the angular rate of change is larger for wet surfaces than for dry surfaces.

The proposed expression for α, derived from our extensive measured data base is of the following form;

$$\alpha = \left[1 - 0.2(\sin\theta)^{A(ks,\Gamma_0)}\right](\cos\theta)^{B(ks,\Gamma_0)} \qquad (7)$$

where $k = 2\pi/\lambda$, s is the rms height, and Γ_0 is the Fresnel reflectivity of the surface at nadir. $A(ks, \Gamma_0)$ and $B(ks, \Gamma_0)$ are chosen such that expression (7) fits best the measured data. The following expressions for A and B are obtained using the backscattered data at 1.5, 4.75, and 9.5 GHz frequencies;

$$\begin{aligned} A(ks,\Gamma_0) &= (16.5\Gamma_0 + 5.6)\exp[-41.6ks\Gamma_0^2] \\ B(ks,\Gamma_0) &= 8.1\Gamma_0 ks \exp[-1.8ks]\,. \end{aligned} \qquad (8)$$

Figures 4 (a)-(c) compare the empirical model with the measured data for surface 1 ($s = 0.4cm$) with $m_v = 0.29$ at three different frequencies. The model agrees well with the measured data at all frequencies. The model was also compared with an independent data set which is measured by a different radar system operated at 1.25 and 5.3 GHz for three different bare soil surfaces ($s1 = 0.78$, $s2 = 1.2$, and $s3 = 4.0cm$, $m_v \approx 0.07$). Again agreement between the proposed model and the measurement data is very good as shown in Fig. 5. Figure 6 shows the correlation of the estimated α using the ground truth data and the empirical model with the measured α using scatterometers. This figure includes all of the data used in development of the empirical model. Figure 7 shows the correlation between the measured α and the estimated α for the independent data set which includes the data for three different surfaces, three incidence angles (30°, 40°, and 50°), and two frequencies (1.25 and 5.3 GHz).

5. CONCLUDING REMARKS

The major results of this study for the phase difference statistics of bare soil surfaces are summarized below:

1. It is observed that the co-polarized phase angle $\phi_c = \phi_{hh} - \phi_{vv}$ has the probability density function which is strongly dependent upon both the target parameters (roughness and moisture content) and the radar parameters (incidence angle and frequency).

2. The cross-polarized phase angle $\phi_x = \phi_{hv} - \phi_{vv}$ is uniformly distributed over [0, 2π]. Hence the cross-polarized phase angle ϕ_x does not contain any information about the random surfaces.

3. An empirical model for the degree of correlation α is proposed and it is shown that the model provides a good agreement with experimental observations made over the ranges $0.1 \leq ks \leq 6$, $2.5 \leq kl \leq 20$, and $0.08 \leq m_v \leq 0.31$.

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[3] Sarabandi K., Y. Oh, and F.T. Ulaby, "Measurement and Calibration of Differential Mueller Matrix of Distributed Targets," *IEEE Trans. Antennas Propagat.*, vol. 40, pp. 1524-1532, Dec. 1992.

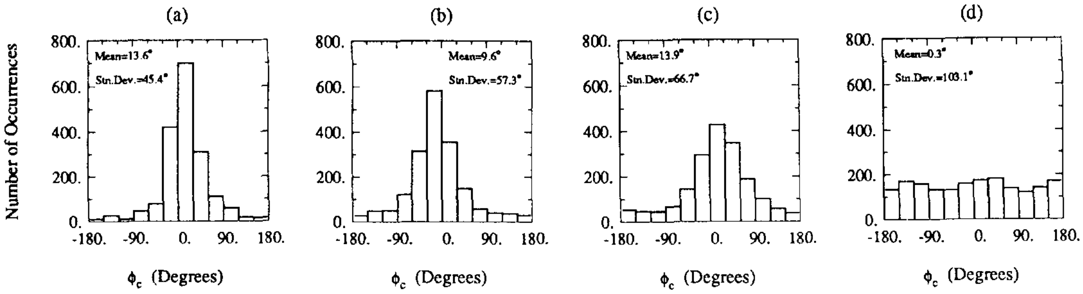

Figure 1: Distribution of co-polarized phase angle $\phi_{hh} - \phi_{vv} = \phi_c$, (a) at 1.5 GHz, (b) at 4.75 GHz, (c) at 9.5 GHz, and (d) cross-polarized phase angle $\phi_{vh} - \phi_{vv} = \phi_x$ at 1.5 GHz, for surface 1 ($s = 0.4cm$) at 30°.

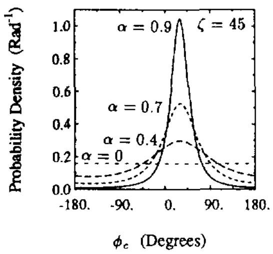

Figure 2: The probability density function of the co-polarized phase angle $\phi_c = \phi_{hh} - \phi_{vv}$ for a fixed value of ζ and four values of α.

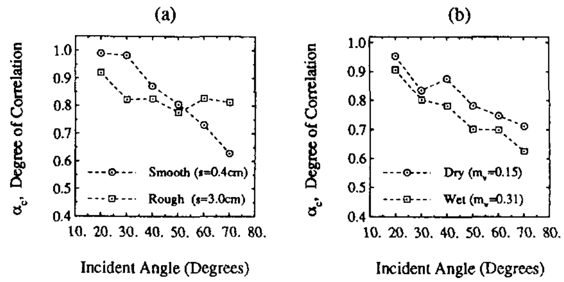

Figure 3: The degree of correlation α of soil surfaces at 4.75 GHz, (a) for two different surface roughnesses ($m_v \approx 0.15$) and (b) for two different moisture conditions ($s = 0.4cm$).

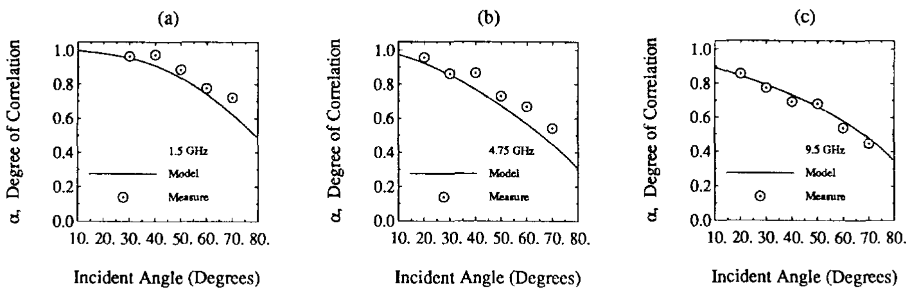

Figure 4: Empirical model compared with the measured data for surface 1 ($s = 0.4cm$) with $m_v = 0.29$ at (a) 1.5 GHz, (b) 4.75 GHz, and (c) 9.5 GHz.

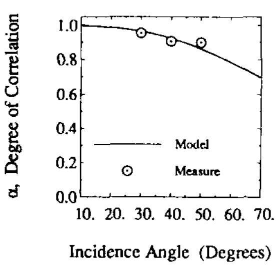

Figure 5: Empirical model compared with an independent data set measured at 5.3 GHz for a surface with $s = 0.78cm$ and $m_v = 0.07$.

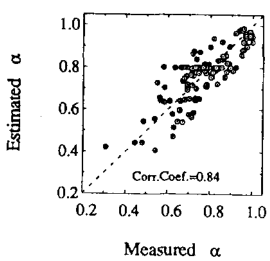

Figure 6: Comparison between the values of α estimated by the proposed model and those measured using scatterometers fot the data set used in development of the model.

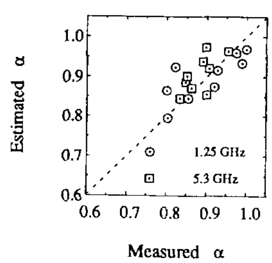

Figure 7: Same as Figure 6 for an independent data set.